

QCD Corrections to Neutron Electric Dipole Moment from Dimension-six Four-Quark Operators

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Abstract

In this Letter, the renormalization-group equations for the (flavor-conserving) CP-violating interaction are derived up to the dimension six, including all the four-quark operators, at one-loop level. We apply them to the models with the neutral scalar boson or the color-octet scalar boson which have CP-violating Yukawa interactions with quarks, and discuss the neutron electric dipole moment in these models.

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I. INTRODUCTION

The electric dipole moment (EDM) for neutrons is sensitive to CP violation in physics beyond the standard model (SM) around TeV scale. This is because, while the CP phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix is $O(1)$, the CKM contribution to the neutron EDM is too much suppressed [1] to be observed in near future. (The recent evaluation of the CKM contribution to the neutron EDM is given in Refs. [2].) The naturalness problem in the Higgs-boson mass term in the SM might require new physics at TeV scale, and many extensions of the SM generically have CP-violating interactions. The supersymmetric standard model, which is the leading candidate for the TeV-scale physics, is severely constrained from the EDM measurements [3] .

The (flavor-conserving) CP-violating effective operators at parton level up to the dimension six are the QCD theta term, the EDMs and the chromoelectric dipole moments (CEDMs) of quarks, the Weinberg's three-gluon operator [4] and the four-quark operators. In the evaluation of the neutron EDM, the CP-violating four-quark operators tend to be ignored since the four-light-quark operators suffer from chiral suppression in many models. However, the four-quark operators including heavier ones, such as bottom/top quarks, may give sizable contributions to the neutron EDM. The EDMs, CEDMs, and the three-gluon operator are radiatively generated from the four-quark operators by integrating out heavy quarks.

In the multi-Higgs models, the Barr-Zee diagrams are known to give the sizable contribution to the neutron EDM [5]. In the Barr-Zee diagrams the heavy-quark loops are connected to light-quark external lines by the neutral scalar boson exchange so that the CEDMs for light quarks are generated at two-loop level at $O(\alpha_s)$. However, it is not clear which renormalization scale should be chosen for α_s . In addition, the contributions from the Barr-Zee diagrams at two-loop level to the quark EDMs vanish at $O(\alpha_s)$. However, it is still unclear that the higher-order corrections to the quark EDMs are negligible in the neutron EDM evaluation.

In this Letter, in order to answer those questions, we derive the renormalization-group equations (RGEs) for the Wilson coefficients for the CP-violating effective operators up to the dimension six at one-loop level, including operator mixing. The RGEs for the EDMs and CEDMs for quarks and the three-gluon operator have been derived in Ref. [6–8]. The next-leading order corrections to them are also partially included [9]. We include the four-quark operators in the calculation at the leading order. Using the derived RGEs, we evaluate the EDMs and CEDMs for light quarks and the three-gluon operators induced by the neutral scalar boson exchange including the QCD correction. We also discuss the four-quark operators induced by the color-octet scalar boson.

This Letter is organized as follows. In the next section, we review the neutron EDM evaluation from the parton-level effective Lagrangian at the hadron scale. In Section 3, we derive RGEs for the Wilson coefficients for the CP-violating effective operators up to the dimension six at one-loop level. In Section 4, we show the effect of the running α_s on the evaluation of the Wilson coefficients, assuming the neutral scalar boson exchange induces the CP-violating effective operators. In Section 5, another example of the application of the RGEs is shown, assuming the effective operators induced by a color-octet scalar boson. Section 6 is devoted to conclusion.

II. NEUTRON EDMS

First, we review about evaluations of the neutron EDM from the low-energy effective Lagrangian at parton level. The CP-violating interaction at parton level around the hadron scale ($\mu_H = 1 \text{ GeV}$) is given by

$$\begin{aligned} \mathcal{L}_{\text{CPV}} = & \theta \frac{\alpha_s}{8\pi} G_{\mu\nu}^A \tilde{G}^{A\mu\nu} \\ & - \frac{i}{2} \sum_{q=u,d,s} d_q \bar{q}(F \cdot \sigma) \gamma_5 q - \frac{i}{2} \sum_{q=u,d,s} \tilde{d}_q \bar{q} g_s (G \cdot \sigma) \gamma_5 q \\ & + \frac{1}{3} w f_{ABC} G_{\mu\nu}^A \tilde{G}^{B\nu\lambda} G_\lambda^{C\mu}. \end{aligned} \quad (1)$$

Here, $F_{\mu\nu}$ and $G_{\mu\nu}^A$ ($A = 1-8$) are the electromagnetic and gluon field strength tensors, g_s is the strong coupling constant ($\alpha_s = g_s^2/4\pi$), $F \cdot \sigma \equiv F_{\mu\nu} \sigma^{\mu\nu}$, $G \cdot \sigma \equiv G_{\mu\nu}^A \sigma^{\mu\nu} T^A$, and $\tilde{G}_{\mu\nu}^A \equiv \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^{A\rho\sigma}$ with $\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$ and $\epsilon^{0123} = +1$. The matrix T^A denotes the generators in the $\text{SU}(3)_C$ algebra, and f^{ABC} is the structure constant. The first, second, third and forth terms in Eq. (1) are called the QCD θ term, the EDM and the CEDM for quarks, and the three-gluon operator, respectively. In this Letter, the covariant derivative is defined as $D_\mu = \partial_\mu - ieQ_q A_\mu - ig_s G_\mu^A T^A$, in which A_μ and G_μ^A are gauge fields for $\text{U}(1)_{EM}$ and $\text{SU}(3)_C$, respectively with Q_q , the QED charge ($(Q_u, Q_d, Q_s) = (2/3, -1/3, -1/3)$). In Eq. (1), we ignore the CP-violating four-quark operators, since their coefficients are often proportional to the light-quark masses in typical models, as mentioned in the Introduction.

The neutron EDM is evaluated from the low-energy interaction at parton level with the naive dimensional analysis, the chiral perturbation theory, and the QCD sum rules, though they are considered to have large uncertainties. The evaluation in term of the QCD sum rules is more systematic than the others, at least for the contributions from the QCD theta term, and the quark EDMs and CEDMs to the neutron EDM [10]. The recent evaluation of the neutron EDM with the

QCD sum rules [11] is

$$d_n \simeq 2.9 \times 10^{-17} \bar{\theta} [e \text{ cm}] + 0.32d_d - 0.08d_u + e(+0.12\tilde{d}_d - 0.12\tilde{d}_u - 0.006\tilde{d}_s) . \quad (2)$$

In the evaluation, the recent QCD lattice result is used for the low-energy constant λ_n , which is defined by $\langle 0 | \eta_n(x) | N(\vec{p}, s) \rangle = \lambda_n u_n(\vec{p}, s)$ with $\eta_n(x)$ the neutron-interpolating field. If a value of λ_n evaluated with the QCD sum rules is used, the neutron EDM is enhanced by about five times compared with Eq. (2).

The contribution from the three-gluon operator might be comparable to the quark EDMs and CEDMs. The quark EDMs and CEDMs are proportional to the quark masses, while the three-gluon operator does not need to suffer from chirality suppression. However, the size of the contribution from the three-gluon operator depends on the methods of the evaluation. In Ref. [12] the authors compare the several evaluations and propose

$$d_n(w) \sim (10 - 30) \text{ MeV} \times ew . \quad (3)$$

III. OPERATOR BASES AND ANOMALOUS DIMENSION MATRIX

We would like to introduce heavy quarks in the low-energy effective theory and evaluate their contributions to the neutron EDM. In this section, we show the one-loop RGEs for the Wilson coefficients for the CP-violating effective operators up to the dimension six, including heavy quarks.

First, we define the operator bases for the RGE analysis. The flavor-conserving effective operators for the CP violation in QCD are given up to the dimension six as

$$\begin{aligned} \mathcal{L}_{\text{CPV}} = & \sum_{i=1,2,4,5} \sum_q C_i^q(\mu) \mathcal{O}_i^q(\mu) + C_3(\mu) \mathcal{O}_3(\mu) \\ & + \sum_{i=1,2} \sum_{q' \neq q} \tilde{C}_i^{q'q}(\mu) \tilde{\mathcal{O}}_i^{q'q}(\mu) + \frac{1}{2} \sum_{i=3,4} \sum_{q' \neq q} \tilde{C}_i^{q'q}(\mu) \tilde{\mathcal{O}}_i^{q'q}(\mu) , \end{aligned} \quad (4)$$

where the sum of q runs not only light quarks but also heavy ones, and we ignore the QCD theta term since it is irrelevant to our discussion here ¹. The effective operators are defined as

$$\begin{aligned} \mathcal{O}_1^q &= -\frac{i}{2} m_q \bar{q} e Q_q (F \cdot \sigma) \gamma_5 q , \\ \mathcal{O}_2^q &= -\frac{i}{2} m_q \bar{q} g_s (G \cdot \sigma) \gamma_5 q , \\ \mathcal{O}_3 &= -\frac{1}{6} g_s f^{ABC} \epsilon^{\mu\nu\rho\sigma} G_{\mu\lambda}^A G_{\nu}^B G_{\rho\sigma}^C , \end{aligned} \quad (5)$$

¹ The QCD theta term does not contribute to the RGEs for other CP-violating terms. Furthermore, there may be contribution to the QCD theta term from other CP violation terms, while the QCD theta term vanishes dynamically if the Peccei-Quinn symmetry is invoked.

and

$$\begin{aligned}
\mathcal{O}_4^q &= \overline{q}_\alpha q_\alpha \overline{q}_\beta i \gamma_5 q_\beta, \\
\mathcal{O}_5^q &= \overline{q}_\alpha \sigma^{\mu\nu} q_\alpha \overline{q}_\beta i \sigma_{\mu\nu} \gamma_5 q_\beta, \\
\tilde{\mathcal{O}}_1^{q'q} &= \overline{q}'_\alpha q'_\alpha \overline{q}_\beta i \gamma_5 q_\beta, \\
\tilde{\mathcal{O}}_2^{q'q} &= \overline{q}'_\alpha q'_\beta \overline{q}_\beta i \gamma_5 q_\alpha, \\
\tilde{\mathcal{O}}_3^{q'q} &= \overline{q}'_\alpha \sigma^{\mu\nu} q'_\alpha \overline{q}_\beta i \sigma_{\mu\nu} \gamma_5 q_\beta, \\
\tilde{\mathcal{O}}_4^{q'q} &= \overline{q}'_\alpha \sigma^{\mu\nu} q'_\beta \overline{q}_\beta i \sigma_{\mu\nu} \gamma_5 q_\alpha.
\end{aligned} \tag{6}$$

Here, m_q are masses for quark q . In Eq. (6) we explicitly show the color indices, α and β . A factor of $1/2$ appears in front of the fourth term of Eq. (4), since the term is symmetric under the exchange of q' and q . The Wilson coefficients in Eq. (4) are related to the parameters in Eq. (1) as

$$\begin{aligned}
d_q &= m_q e Q_q C_1^q(\mu_H), \\
\tilde{d}_q &= m_q C_2^q(\mu_H), \\
w &= -\frac{1}{2} g_s C_3(\mu_H).
\end{aligned} \tag{7}$$

The RGEs for the Wilson coefficients of these operators are given as follows,

$$\mu \frac{\partial}{\partial \mu} \mathbf{C} = \mathbf{C} \mathbf{\Gamma}, \tag{8}$$

where the Wilson coefficients are written in a column vector as

$$\mathbf{C} = (C_1^q, C_2^q, C_3, C_4^q, C_5^q, \tilde{C}_1^{q'q}, \tilde{C}_2^{q'q}, \tilde{C}_1^{qq'}, \tilde{C}_2^{qq'}, \tilde{C}_3^{q'q}, \tilde{C}_4^{q'q}). \tag{9}$$

The anomalous dimension matrix is calculated at one-loop level as

$$\mathbf{\Gamma} = \begin{bmatrix} \frac{\alpha_s}{4\pi} \gamma_s & \mathbf{0} & \mathbf{0} \\ \frac{1}{(4\pi)^2} \gamma_{sf} & \frac{\alpha_s}{4\pi} \gamma_f & \mathbf{0} \\ \frac{1}{(4\pi)^2} \gamma'_{sf} & \mathbf{0} & \frac{\alpha_s}{4\pi} \gamma'_f \end{bmatrix}, \tag{10}$$

where

$$\gamma_s = \begin{bmatrix} +8C_F & 0 & 0 \\ +8C_F & +16C_F - 4N & 0 \\ 0 & +2N & N + 2n_f + \beta_0 \end{bmatrix}, \tag{11}$$

$$\gamma_f = \begin{bmatrix} -12C_F + 6 & +\frac{1}{N} - \frac{1}{2} \\ +\frac{48}{N} + 24 & +4C_F + 6 \end{bmatrix}, \tag{12}$$

$$\gamma'_f = \begin{bmatrix} -12C_F & 0 & 0 & 0 & +\frac{1}{N} & -1 \\ -6 & +\frac{6}{N} & 0 & 0 & -\frac{1}{2} & -C_F + \frac{1}{2N} \\ 0 & 0 & -12C_F & 0 & +\frac{1}{N} & -1 \\ 0 & 0 & -6 & +\frac{6}{N} & -\frac{1}{2} & -C_F + \frac{1}{2N} \\ +\frac{24}{N} & -24 & +\frac{24}{N} & -24 & +4C_F & 0 \\ -12 & -24C_F + \frac{12}{N} & -12 & -24C_F + \frac{12}{N} & +6 & -8C_F - \frac{6}{N} \end{bmatrix}, \quad (13)$$

$$\gamma_{sf} = \begin{bmatrix} +4 & +4 & 0 \\ -32N - 16 & -16 & 0 \end{bmatrix}, \quad (14)$$

and

$$\gamma'_{sf} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -16N \frac{m_{q'}}{m_q} \frac{Q_{q'}}{Q_q} & 0 & 0 \\ -16 \frac{m_{q'}}{m_q} \frac{Q_{q'}}{Q_q} & -16 \frac{m_{q'}}{m_q} & 0 \end{bmatrix}, \quad (15)$$

where $C_F = (N^2 - 1)/(2N)$ is the Casimir constant of the fundamental representation, $N (= 3)$ is the number of the color, n_f is the number of light flavor quarks, and $\beta_0 (= 11/3 \times N - 2/3 \times n_f)$ is the leading-order beta function of strong coupling constant.

The anomalous dimensions for the dimension-five operators are calculated in Ref. [6], and that for the three-gluon operator is calculated by Ref. [7]. The mixings among the dimension-five operators and the three-gluon operator are found in Ref. [8]. We newly calculate other terms in the anomalous dimensions matrix, which are related with the four-quark operators. We note that the operator mixings between the EDM or CEDM operators and the four-quark operators are generated at $\mathcal{O}(\alpha_s^0)$.

IV. NEUTRAL SCALAR BOSON EXCHANGE

In multi-Higgs models, a color-singlet neutral scalar boson ϕ may have the CP-violating Yukawa coupling with quarks. If the Yukawa interaction violates the CP invariance, the CP-violating four-

quark operators are induced at tree level, after integrating the neutral scalar boson out, as

$$\begin{aligned} C_4^q &= \sqrt{2}G_F \frac{m_q^2}{m_\phi^2} f_S^q f_P^q, \\ \tilde{C}_1^{q'q} &= \sqrt{2}G_F \frac{m_q m_{q'}}{m_\phi^2} f_S^q f_P^{q'}, \\ \tilde{C}_1^{qq'} &= \sqrt{2}G_F \frac{m_q m_{q'}}{m_\phi^2} f_S^{q'} f_P^q, \end{aligned} \quad (16)$$

where we assume that ϕ is heavier than heavy quarks ($m_\phi \gg m_q, m_{q'}$). Here, f_S^q and f_P^q are the CP-even and odd Yukawa coupling constants, respectively, defined as

$$\mathcal{L}_\phi = 2^{1/4} G_F^{1/2} m_q \bar{q}_\alpha (f_S^q + i f_P^q \gamma_5) q_\alpha \phi, \quad (17)$$

where ϕ is a (CP-even) real scalar field, and G_F is the Fermi constant. We parametrize the Yukawa coupling constants as they are proportional to the quark masses. In typical new physics models, these Yukawa coupling constants are taken to be proportional to masses of quarks in order to avoid the stringent experimental constraints from the flavor physics data. For the SM Higgs boson, the Yukawa coupling constants are of $f_S^q = 1$ and $f_P^q = 0$. For the multi-Higgs models, *e.g.*, two-Higgs-doublet models, the coefficients $f_{S/P}^q$ may be much larger than unity because of the enhancement factor originated from the ratio of vacuum expectation values.

It is known that, in these models, the EDMs and CEDMs for light quarks are generated by the Barr-Zee diagrams at two-loop level, and the three-gluon operator is also induced by the heavy-quark loops at two-loop level. Let us derive those contributions using the RGEs for the Wilson coefficients.

In the leading-logarithmic approximation, since C_4^q is non-zero, the coefficients for the EDM and the CEDM operators are generated from the one-loop RGEs as

$$C_1^q = C_2^q = -\frac{1}{4\pi^2} C_4^q \ln \frac{m_\phi}{m_q}. \quad (18)$$

These contributions are compared with the explicit calculation of one-loop diagrams with the scalar boson exchange as

$$C_1^q = C_2^q = -\frac{1}{4\pi^2} C_4^q \left(\ln \frac{m_\phi}{m_q} - \frac{3}{4} \right), \quad (19)$$

where a limit of $m_q \ll m_\phi$ is taken. The second term in the parentheses should be considered as the short-distance contribution in which the loop momentum is around m_ϕ .

The one-loop contributions from C_4^q to the EDMs and CEDMs for light quarks ($q = u, d, s$) are negligible in the neutron EDM since they are suppressed by powers of the light-quark masses.

However, when the CEDMs for heavy quarks are generated, the three-gluon operator is induced by integration of heavy quarks as follows [13],

$$C_3(m_q) = \frac{\alpha_s(m_q)}{8\pi} C_2^q(m_q). \quad (20)$$

Thus, from Eqs. (19, 20), we get

$$C_3 = -\frac{\alpha_s}{32\pi^3} C_4^q \left(\ln \frac{m_\phi}{m_q} - \frac{3}{4} \right). \quad (21)$$

The result is consistent with the explicit calculation of the two-loop diagrams for the three-gluon operator [14].

When $\tilde{C}_1^{q'q}$ and/or $\tilde{C}_1^{qq'}$ are non-zero, the contribution to the CEDMs for light quarks is derived at the two-loop level, using the RGEs for the the Wilson coefficients, Eq. (10), as

$$C_2^q = \frac{\alpha_s}{8\pi^3} \frac{m_{q'}}{m_q} \left(\ln \frac{m_\phi}{m_{q'}} \right)^2 \left[\tilde{C}_1^{q'q} + \tilde{C}_1^{qq'} \right]. \quad (22)$$

This is because $\tilde{C}_1^{q'q}$ and $\tilde{C}_1^{qq'}$ are mixed with $\tilde{C}_4^{q'q}$ and $\tilde{C}_4^{qq'}$ which induce the CEDMs. When the Yukawa coupling constants in Eq. (17) are proportional to the quark masses, the induced CEDMs for light quarks are not suppressed by their masses, compared with the one-loop contribution. The result in Eq. (22) is consistent with the explicit calculation of the Barr-Zee diagrams [5] in a limit of $m_\phi \gg m_q, m_{q'}$. It is known that the contribution to the neutron EDM from the Barr-Zee diagram at two-loop can be competitive with those from the one-loop diagrams [5]. Therefore, the four-quark operator for heavy quarks can be important in some cases.

On the other hand, the EDMs for light quarks are not generated by two-loop level diagrams of $O(\alpha_s)$, even if $\tilde{C}_1^{q'q}$ and $\tilde{C}_1^{qq'}$ are non-zero. The EDMs for light quarks have contributions from $\tilde{C}_3^{q'q}$ ($\tilde{C}_3^{qq'}$) and $\tilde{C}_4^{q'q}$ ($\tilde{C}_4^{qq'}$) in Eq. (10). Their contributions are exactly canceled with each others so that the EDMs vanish at the order. This is also consistent with the explicit calculation of the Barr-Zee diagrams. The EDMs generated at two-loop level are suppressed by α . However, the running effect of the strong coupling constant may prevent the cancellation so that the EDM would be enhanced.

Now, let us consider the effect of the running strong coupling constant, $\alpha_s(\mu)$. Here we compare values of the EDM and CEDM operators for down quark and the three-gluon operators including and not including the renormalization-group evolution of the strong coupling constant. We assume that the Yukawa coupling constants for down and bottom quarks with ϕ are non-zero in Eq. (17)

and then

$$\begin{aligned}\tilde{C}_1^{bd}(m_\phi) &\neq 0, \quad C_4^b(m_\phi) \neq 0, \\ C_1^b(m_\phi) &= C_2^b(m_\phi) = +\frac{3}{16\pi^2}C_4^b(m_\phi).\end{aligned}\tag{23}$$

The last assumption comes from Eq. (19).

In Fig. 1 the CEDM for down quark, \tilde{d}_d , (a) and the coefficient of the three-gluon operators, w , (b) at the hadron scale ($\mu = \mu_H = 1$ GeV) are shown as functions of m_ϕ with $f_S^d = f_P^d = 1$ and $f_S^b = f_P^b = 1$. Here, we ignore the contributions from top quark, and other short-distance effects. If the scalar mass m_ϕ is larger than the top quark mass ($m_\phi > m_t$), the RGEs are solved using β_0 with $n_f = 6$, or if not, with $n_f = 5$. When bottom quark is integrated out, the Wilson coefficient of Weinberg operator emerges. Then the RGEs are solved using β_0 with $n_f = 4$ to the scale $\mu = m_c$, and with $n_f = 3$ to the scale $\mu = 1$ GeV. We use $m_d(\mu_H) = 9$ MeV, $m_c(m_c) = 1.27$ GeV, $m_b(m_b) = 4.25$ GeV, $m_t(m_t) = 172.9$ GeV, and $\alpha_s(m_Z) = 0.12$. For the coefficient w , we multiply 10 MeV in the figure, which is a factor in Eq. (3), so that one may estimate the contribution to the neutron EDM. It is from Eqs. (2,3) found that the three-gluon operator might be comparable to the CEDM when $f_{S/P}^d \sim f_{S/P}^b$.

In Fig. 2 the ratios of the CEDM for down quark (a) and the three-gluon operator (b) at $\mu = m_b$ between including the running effect of α_s and not including it (using the constant coupling $\alpha_s = \alpha_s(m_b)$), are shown as functions of m_ϕ . It is found that the running coupling $\alpha_s(\mu)$ changes the CEDM by about 20% while the three-gluon operator is changed by at most 10 %. These results come from inclusions of the four-quark operators to the RGEs for the Wilson coefficients.

In Fig. 3 the ratio of the EDM and CEDM for down quark is presented as a function of m_ϕ . The non-zero value of the EDM is generated as we mentioned. It is also found that the ratio is roughly proportional to $\log m_\phi$, and the absolute value is about 0.14. In the evaluation of the neutron EDM with the QCD sum rules, the size of the contribution from the down-quark EDM is 30–40 % of that from the CEDM.

V. COLOR-OCTET SCALAR BOSON EXCHANGE

In the previous section it is shown that the neutral scalar boson does not generate sizable EDMs for light quarks via two-loop diagrams at $O(\alpha_s)$. This comes from cancellation between contributions via $\tilde{C}_3^{q'q}$ ($\tilde{C}_3^{qq'}$) and $\tilde{C}_4^{q'q}$ ($\tilde{C}_4^{qq'}$). This situation is different when the scalar boson has color. Now let us assume that a color-octet scalar boson $\Sigma(= \Sigma^A T^A)$ is introduced and it has

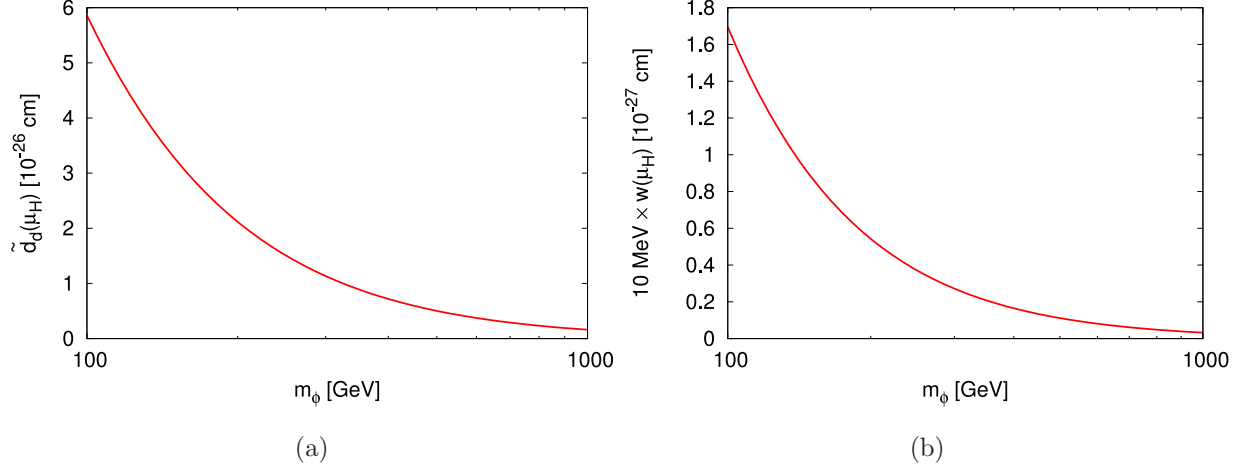


FIG. 1: (a) CEDM for down quark, \tilde{d}_d , and (b) coefficient of three-gluon operator, w , at hadron scale as functions of m_ϕ .

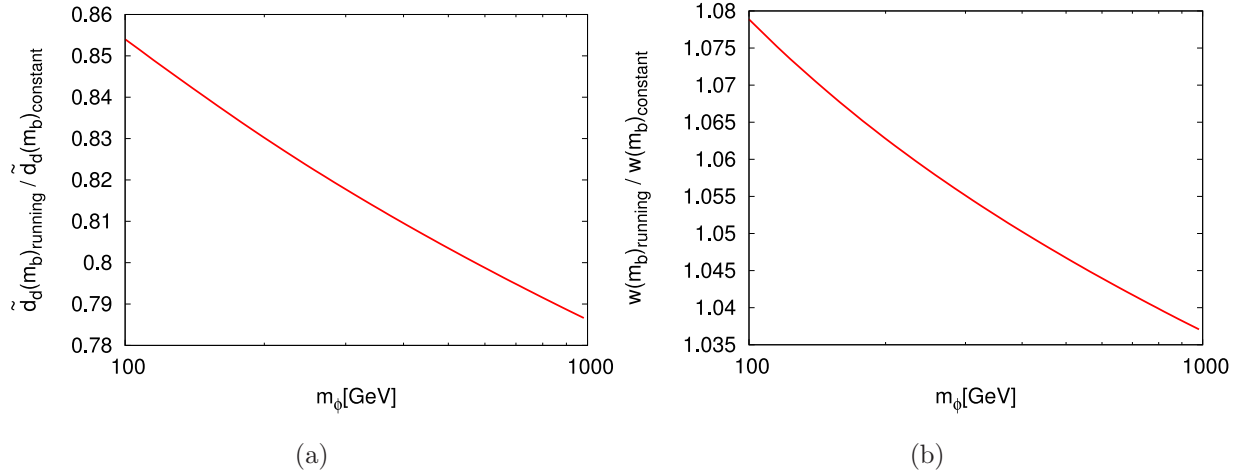


FIG. 2: (a) : Ratio of the CEDM for down quark, \tilde{d}_d , at $\mu = m_b$ between including and not including running of the strong coupling constant, as a function of m_ϕ . (b) The same ratio for coefficient of three-gluon operator, w .

CP-violating Yukawa interactions with quarks,

$$\mathcal{L}_\Sigma = 2^{1/4} G_F^{1/2} m_q \bar{q}_\alpha (f_S^q + i f_P^q \gamma_5) q_\beta \Sigma_{\alpha\beta}. \quad (24)$$

The octet scalar fields may appear in new physics beyond the SM such as a radiative seesaw model for the neutrino masses [15], and a grand unified model [16].

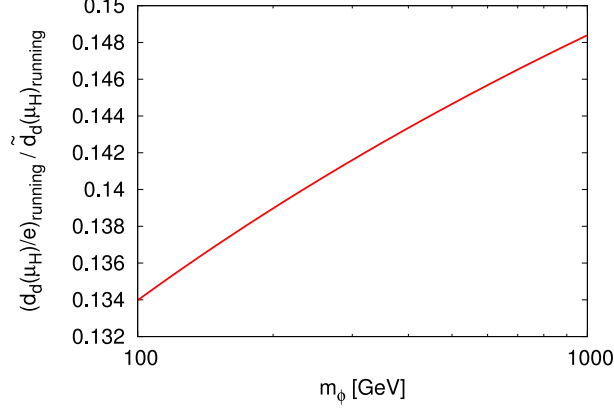


FIG. 3: Ratio of EDM and CEDM for down quark, $(d_d/e)/\tilde{d}_d$, at hadron scale as a function of m_ϕ .

Integration of the Σ leads to the following Wilson coefficients,

$$\begin{aligned}
C_4^q &= -\sqrt{2}G_F \frac{m_q^2}{m_\Sigma^2} \left(\frac{1}{4} + \frac{1}{2N} \right) f_S^q f_P^q, & C_5^q &= -\sqrt{2}G_F \frac{m_q^2}{m_\Sigma^2} \frac{1}{16} f_S^q f_P^q, \\
C_1^{q'q} &= -\sqrt{2}G_F \frac{m_q m_{q'}}{m_\Sigma^2} \frac{1}{2N} f_S^{q'} f_P^q, & C_2^{q'q} &= \sqrt{2}G_F \frac{m_q m_{q'}}{m_\Sigma^2} \frac{1}{2} f_S^{q'} f_P^q, \\
C_1^{qq'} &= -\sqrt{2}G_F \frac{m_q m_{q'}}{m_\Sigma^2} \frac{1}{2N} f_S^q f_P^{q'}, & C_2^{qq'} &= \sqrt{2}G_F \frac{m_q m_{q'}}{m_\Sigma^2} \frac{1}{2} f_S^q f_P^{q'}.
\end{aligned} \tag{25}$$

Now $C_2^{qq'}$ and/or $C_2^{q'q}$ are generated. At the two-loop level, the EDMs for light quarks have contributions via $\tilde{C}_3^{q'q}$ ($\tilde{C}_3^{qq'}$) and $\tilde{C}_4^{q'q}$ ($\tilde{C}_4^{qq'}$) with different weights. This is different from the case of the neutral scalar boson exchange as discussed in the previous section. The EDMs for light quarks are generated at two-loop level of $O(\alpha_s)$ as

$$C_1^q = -\frac{\alpha_s}{4\pi^3} \frac{m_{q'}}{m_q} \frac{Q_{q'}}{Q_q} C_F \left(\ln \frac{m_\Sigma}{m_{q'}} \right)^2 \left[\tilde{C}_2^{q'q} + \tilde{C}_2^{qq'} \right], \tag{26}$$

which is compared with the CEDMs for light quarks as

$$C_2^q = \frac{\alpha_s}{8\pi^3} \frac{m_{q'}}{m_q} \left(\ln \frac{m_\Sigma}{m_{q'}} \right)^2 \left[-\tilde{C}_1^{q'q} + \left(\frac{1}{2N} - C_F \right) \tilde{C}_2^{q'q} - \tilde{C}_1^{qq'} + \left(\frac{1}{2N} - C_F \right) \tilde{C}_2^{qq'} \right]. \tag{27}$$

In Fig. 4 the EDM and CEDM for down quark at the hadron scale are shown as functions of m_Σ with $f_S^d = f_P^d = 1$ and $f_S^b = f_P^b = 1^2$. Here we use the running coupling for α_s . It is found that the EDM contribution is larger than the CEDM ones in the neutron EDM when we adopt the QCD sum rule result on the neutron EDM evaluation. (See Eq. (2).)

For completeness, we also show the magnitude of the three-gluon operator in Fig. 5. Here, we ignore the short-distance contributions to the CEDMs for heavy quarks and three-gluon operator

² If the electrically charged color-octet scalar boson exists, we may have large contributions from the top quark [16].

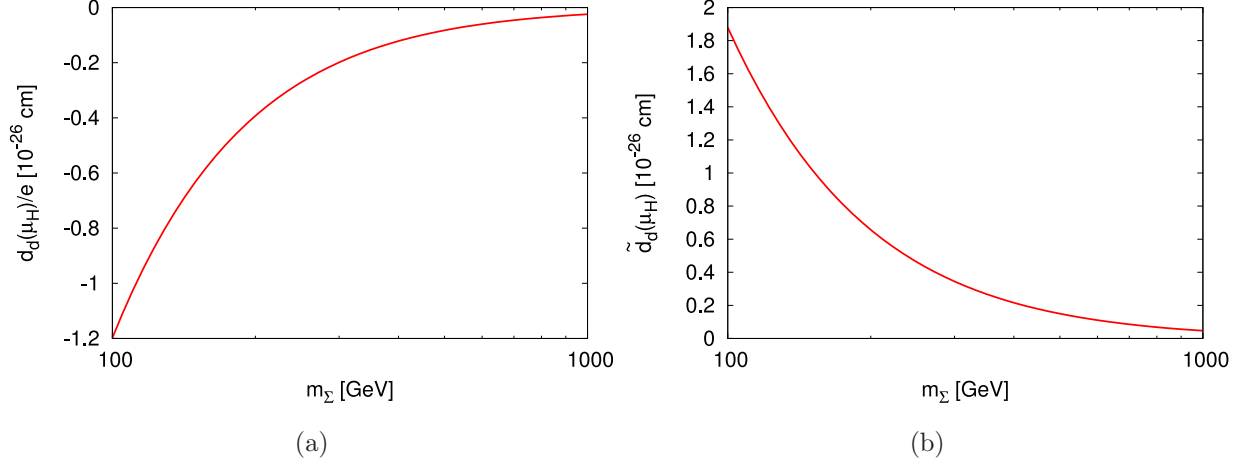


FIG. 4: (a) EDM d_d/e and (b) CEDM \tilde{d}_d for down quark at hadron scale as functions of m_Σ .

whose loop momenta are around m_Σ and take $C_2^q(m_\Sigma) = C_3(m_\Sigma) = 0$ for simplicity³. Again, it is found that the three-gluon operator might give a comparable effect to other contributions.

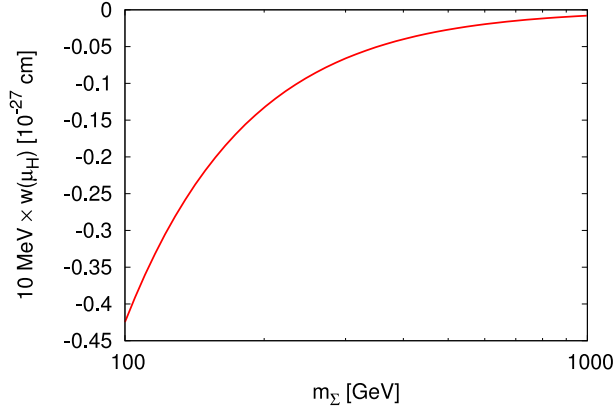


FIG. 5: Three-gluon operator w at hadron scale as a function of m_Σ .

VI. CONCLUSION

In this Letter, we have derived the renormalization-group equations for the CP-violating interaction including the quark EDMs and CEDMs and the Weinberg's three-gluon operator as well as all the flavor-conserving four fermion operators. The operator mixings between the (C)EDM operators and the four-quark operators are arisen at the order of α_s^0 , which can give large contributions

³ The short-distance contribution to the three-gluon operator could come from a diagram in which three gluons are emitted from each quark and scalar boson line, and the evaluation is beyond the scope of this work.

to the EDMs via the renormalization-group evolution.

Assuming the CP-violating Yukawa interactions for the neutral scalar bosons, it is known that the CEDMs for light quarks are generated from the diagrams with heavy-quark loops, called as the Barr-Zee diagrams. We show that when the neutral scalar boson is much heavier than heavy quarks, the Barr-Zee diagrams are systematically evaluated with the RGEs of the CP-violating interaction. We also show that the running effect of the strong coupling constant gives corrections to the contribution with more than 20 % compared with assuming the constant coupling. The uncertainties in the calculation of the neutron EDM have been estimated in the literature [17]. It gives about 50 % error for the QCD sum rule, while 40 % error for the low-energy constant evaluated from the lattice QCD calculation. Therefore, hadronic uncertainties would overcome the QCD corrections from the renormalization-group evolution at this moment. We hope that the lattice QCD simulation will improve and reduce uncertainties significantly [18].

The Barr-Zee diagrams at two-loop level do not contribute to the EDMs for light quarks at $O(\alpha_s)$. We show using the RGEs of the CP-violating interaction of a color-singlet scalar boson with quarks that ratio of the quark EDM over the CEDM is about 0.14, and it is roughly proportional to $\log m_\phi$. Thus, the contribution is not negligible to the neutron EDM at all. When the color-octet scalar boson has CP-violating Yukawa interaction with quarks, the quark EDMs are generated at two-loop level, and they are comparable to the quark CEDMs.

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